**EDCI 564 – Technology Integration Project**

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**Chapter 5: Graphing the Trigonometric Functions**

**Lesson Plan**

**Original Instructional Unit:**

 I will be working with Abeka’s precalculus textbook. While I have taught precalculus before, I used Saxon’s GTA (geometry, trigonometry, and advanced algebra), so this is a new textbook and edition for me. Abeka has been modifying its curriculum to add more technology. Still, this text has not been modified yet, so I need to adjust these lessons to implement more technology personally.

 The original chapter, teacher lesson plan, quizzes, test and other supplementary materials can be accessed with this link to my Google Drive.

[Murray Technology Integration Project Attachments](https://drive.google.com/drive/folders/19otLP6k2JsxcuuJexzD_Cv8CvwaMID-A?usp=drive_link)

**SAMR Worksheet**

|  |  |
| --- | --- |
| **Redefinition**(new tasks, previously inconceivable) | Connecting the unit circle to the graphs of the trigonometric functions and then realizing that they repeat can be difficult for students to see in their heads. I have tried to draw pictures of the relationships on the board for the class, but being a mathematician instead of an artist has made it harder for students to see. An online interactive unit circle shows the sine, cosine, and tangent values on the unit circle. Those values are also represented as reference triangles and then translated to a graph that will also show the repetitive or periodic nature of the graphs. This interactive graph puts so many relationships together that I can now show with motion that I would not have been able to do with drawings on a board.[Interactive Unit Circle](https://www.mathsisfun.com/algebra/trig-interactive-unit-circle.html) |
| **Modification**(significant task redesign) | Applying sine and cosine graphs in periodic motion can be difficult. The textbook shows some basic diagrams with arrows. But with more visual learners in the classroom, students need to see something more tangible and maybe even do experiments.I teach in a small school, and no money is devoted to lab equipment. YouTube videos allow me to demonstrate the application of sine and cosine graphs. These videos will help the students see the back-and-forth motion that occurs when a pendulum or spring is set in motion. There is a series of short clips of periodic motion that I can pick from to show my students, and a video on simple harmonic frequencies so the students can also hear the difference.[YouTube - Periodic Motion Series](https://www.youtube.com/playlist?list=PLLmvDJR_FuZKd_2nwR8YY_mvpBIwww5sB)[YouTube - Simple Harmonic Motion Experiments](https://www.youtube.com/playlist?list=PLLmvDJR_FuZKd_2nwR8YY_mvpBIwww5sB) |
| **Augmentation**(functional improvement) | Graphing by hand can be time-consuming for students and limits the pattern or shape of the graph to the rough sketch that the student has completed. Graphing calculators can make graphing better but can be expensive, especially for students who only need the graphing calculator for high school.With using an online graphing calculator like Desmos, students can access a graphing calculator without the cost. Desmos will even work with mobile devices so students can use a computer, phone, or tablet to complete their homework. Desmos allows students to graph multiple equations at the same time and color code the equations to match the graph to its equation. With Desmos, users can zoom in and zoom out on the graph to see more of the graph and allow students to detect patterns more easily.[Desmos](https://www.desmos.com/calculator) |
| **Substitution**(no functional change) | Grading quizzes can be time-consuming for the teacher, and if the test is coming up students need quick feedback to prepare for the test. Online quizzes would allow teachers to get grades and feedback to the students faster. [FlexiQuiz](https://www.flexiquiz.com/) |
| **Unit/Lesson** | The current unit lesson plans do not use technology at all. Everything is completed with paper and pencil.Students work with textbooks, pencil, and paper. There is no need for any use of technology. The teacher may choose to use PowerPoints for the lecture information and terms for the students to take notes on. However, students are to write down notes of the example problems and other worked-out problems that are in the textbook which the teacher decides to demonstrate. Homework is completed on notebook paper and turned in. There are a few quizzes interspersed throughout the chapter. Quizzes tend to cover 2-3 sections and are done on paper on which the students would also graph or receive a graph and pick out the appropriate parts. A test is given at the end of the chapter (in 3 weeks) to test for understanding and ability to graph the different trigonometric graphs. |

**Updated Instructional Unit**

**Applied Technology Project**

**Precalculus Chapter 5 Lesson Plan**

**Topic: Precalculus - Graphing the Trigonometric Functions**

**Target Population:**

* Learner Level: K-12, usually students enrolled in a precalculus class are in 11th-12th grade
* Population Characteristics: High school juniors or seniors who are diligent or interested in learning or are interested in continuing to college. Especially for students who are thinking about applied science careers or even trades with electricity, plumbing, and construction.
* Instructional Grouping: This will be for the whole class.

**Curriculum Links:**

Students should have previous knowledge of the trigonometric functions from algebra and geometry courses. Students are also used to graphing linear and quadratic equations from algebra. We will be spiraling from that knowledge and applying graphing to all six trigonometric functions. Graphing will be revisited in Chapter 10 of the textbook as we look at graphing and geometric concepts. There is even an application section where students will graph multiple equations and then find intersections and overlaps that can be applied to motion which students will also see in their physics classes.

**Objectives:**

By the end of the chapter students should be able to do the following:

* Know key terms: intercept, symmetry, mirror image, sinusoid, amplitude, translation, phase shift, asymptote
* Evaluate functions with constants or expressions
* Write inequalities as interval notation and vice versa
* Graph inequalities and interval notation on a number line
* Distinguish between odd and even functions including the six trigonometric functions
* Can show that sine and cosine graphs are periodic functions
* Able to fill in an x-y table for and
* Graph sinusoids by hand and using an online graphing calculator (Desmos)
	+ Including scalings and translations of the graphs
	+ Including the reciprocal functions
* Periodic functions are examples of simple harmonic motion
* Define frequency and know the unit is Hertz (Hz)
* Can calculate the frequency from the period and vice versa
* Can extract information from a word problem scenario to write a function and graph it.

**Materials:**

 **Teacher**

* Teacher edition
* Solution Key
* Computer or Tablet
* Worksheets
* Tests and Quizzes

 **Students**

* Textbook
* Paper, pencil, calculator
* Teacher handouts
* Laptop, tablet, or smartphone (has an internet connection)

**Time:**

 This chapter is designed to be completed in three school weeks or fifteen 50-minute class periods. For schools that are using block scheduling, this could be completed in ten or eleven 70-minute class periods.

**Scope and Sequence:**

 **5.1 – Functional Notation and Interval Notation (1 Day)**

|  |  |
| --- | --- |
| Start of Class: | Review test from Chapter 4 |
| Objectives: | * Know terms
* Evaluate functions with constants or expressions
* Write inequalities as interval notation and vice versa
* Graph inequalities and interval notation on a number line
 |
| Lesson: | * Review functional notation
	+ F(x) is a fancy name for “y”
	+ When evaluating functions, whatever is in the parentheses will be substituted for x.
	+ Work out Example 1
* Discuss domain and range of functions
	+ Domain – input or x-values
	+ Range – output or y-values
	+ Interval notation can be used as an abbreviated way of listing all the values included for the domain or what is possible for the range.
	+ Refer to Table A with the different notations and names for the intervals. Previously we said closed, open, or mixed. There are half-open, bounded, and unbounded that will be used for the mixed intervals.
	+ Work Example 2
* Exercise 5.1
	+ Work problems 1 and 2 together as a class.
	+ Then have students go up to the whiteboards and work on #3-15 odd.
	+ Walk around, help students, and check on their answers.
 |
| Homework Assigned: | Assign homework at the end of class and give students a couple of minutes to get started on it.p. 72 Exercise 5.1 – evensThis is to be done on a separate sheet of paper. |

 **5.2 – Graphing Concepts (1 Day)**

|  |  |
| --- | --- |
| Start of Class: | Have students put solutions from yesterday’s assignment on the board. Students check the work on the board with their solutions. Ask for any specific problems they had while completing the homework that they need in more detail or show another solution. Have students turn in homework. |
| Objectives: | * Distinguish between odd and even functions
* Know sine is an odd function and cosine is an even function
* Know the negative angle identities for sine and cosine
* Can show that sine and cosine graphs are periodic functions
 |
| Lesson: | * Graphing can be completed in many ways
	+ A common way and used previously is the x-y table.
	+ Plug in x-values, calculate y-values, plot the points, and connect the dots with smooth, curved lines
* Even and odd functions
	+ Graphs that mirror themselves (symmetrical) across the y-axis are even functions,
	+ Graphs that are mirrored in the opposite direction about the origin (0,0) are odd functions,
	+ Work examples and show on [Desmos](https://www.desmos.com/)
* Look at sine and cosine graphs and their values.
	+ Sine is odd
	+ Cosine is even
	+ This is called their negative angle identities
	+ Since both graphs repeat the same answers after one rotation of the unit circle, they are called periodic graphs.
	+ When applying these graphs to the phenomenon in nature we call it simple harmonic motion
* Exercise 5.2
	+ Work problems 1, 10, 22, and 34 together as a class.
	+ Then have students go to the whiteboards and work on the odd problems.
	+ Walk around, help students, and check on their answers.
 |
| Homework Assigned: | Assign homework at the end of class and give students a couple of minutes to get started on it.p. 75 Exercise 5.2 – by 4s (complete the divisible problem numbers by 4)This is to be done on a separate sheet of paper. |

 **5.3 – Graphs of Sine and Cosine (2 Days)**

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| --- | --- |
| Start of Class: | Have students put solutions from yesterday’s assignment on the board. Students check the work on the board with their solutions. Ask for any specific problems they had while completing the homework that they need in more detail or show another solution. Have students turn in homework. |
| Objectives: | * Know terms: intercept, symmetry, mirror image, sinusoid, and amplitude.
* Able to fill in an x-y table for and
* Graph sinusoids by hand and using an online graphing calculator (Desmos)
 |
| Lesson: | * Introduce radians
	+ 1 radian is the length of 1 radius on the circumference of the circle.
	+ To complete one full revolution around the circle, you would need approximately 6.28 radians or 3.14 radians to go halfway around the circle.
* Define intercepts, symmetry, and mirror image
	+ Intercept – points when the graph crosses the x or y-axis.
	+ Symmetry – a mirror image created by a line or point
	+ Mirror image – a reflection or point that is the same distance from the line or point of symmetry
* Graph
	+ Using a x-y table
	+ Using Desmos
	+ This is called a sine curve or sinusoid
* Graph
	+ Using a x-y table
	+ Using Desmos
	+ This is also called a sinusoid because it has the same basic shape

Depending on the time left for the day, we may be able to solve a couple of problems from Exercise 5.3. If not, pick it up here the next day.* Exercise 5.3
	+ Work problems 3 and 4 together as a class.
	+ Have students come up to the board to work a couple of the odd problems by hand.
	+ Have the students also get out their electronics to graph using Desmos.
	+ Walk around, help students, and check on their answers.
 |
| Homework Assigned: | Day 1 – No homework but give students a few minutes to work on homework that will be assigned on Day 2.Day 2 - Assign homework at the end of class and give students a couple of minutes to work on it.pp. 79-80 Exercise 5.3 – evens* Each graph should be done by hand
* Email screenshots of graphs on Desmos to the teacher. Please do not complete more than 3 functions on 1 screenshot (minimum of 2 screenshots)
 |

 **5.4 – Vertical and Horizontal Scaling (2 Days)**

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| --- | --- |
| Start of Class: | Have students put solutions from yesterday’s assignment on the board. Students check the work on the board with their solutions. Ask for any specific problems they had while completing the homework that they need in more detail or show another solution. Have students turn in homework.After homework is collected, have students take [Quiz 5.1-5.3](https://www.flexiquiz.com/SC/N/8d0c520b-4f91-4646-a367-b597bc470a76) online. |
| Objectives: | * Know vertical scaling changes the amplitude of a graph
* Know horizontal scaling changes the period of the graph.
 |
| Lesson: | * Vertical scaling
	+ The amplitude is not 1.
	+ The amplitude is represented by the *a* in
	+ If *a* is a fraction, the graph is short. If it is larger than 1 then the graph is tall.
	+ *-a* inverts the graph
	+ Show Example 1 in Desmos
* Horizontal scaling
	+ Changing the period will change the length (wavelength) of the graph until it repeats.
	+ Changing the period will also change the frequency of a graph. Frequency
	+ Show Example 2 in Desmos
* Play YouTube [frequency video](https://www.youtube.com/watch?v=L-q-KFwjwDs)
	+ This video is an application of the changing horizontal scaling of sinusoid functions

Depending on the time left for the day, we may be able to complete a couple of problems from Exercise 5.4. If not, pick it up here the next day.* Exercise 5.4
	+ Work problems 1, 13, and 25 together as a class.
	+ Have students come up to the board to work the odd problems by hand.
	+ Have the students also get out their electronics to graph using Desmos.
	+ Walk around, help students, and check on their answers.
 |
| Homework Assigned: | Day 1p. 84 Exercise 5.4 – multiples of 4 on Desmos* Email screenshots of graphs on Desmos to the teacher. Please do not complete more than 2 functions on 1 screenshot (minimum of 2 screenshots)

Day 2p. 84 Ex 5.4 #26-40 every other event* Graph these by hand
 |

 **5.5 – Horizontal and Vertical Translations and Graphing by Addition of Ordinates (2 days)**

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| --- | --- |
| Start of Class: | Have students put solutions from yesterday’s assignment on the board. Students check the work on the board with their solutions. Ask for any specific problems they had while completing the homework that they need in more detail or show another solution. Have students turn in homework.Go over answers to Quiz 5.1-5.3Day 2 - Give Quiz 5.3-5.4 |
| Objectives: | * Know the terms of translation and phase shift
* Able to graph translations
 |
| Lesson: | * Horizontal and vertical translations
	+ Horizontal translations are also called phase shifts
	+ Translations of graphs keep the same shape of the graph but just shift it to a new starting point.
	+ Work Examples 1 and 2
* Exercise 5.5
	+ Do problems 1 and 13 together as a class.
	+ Have students come to the board and work on the odd problems on p. 88.
	+ Also have the students work on their devices and graph some on Desmos

Day 2* Addition of Ordinate
	+ This is used to combine two functions into one function
	+ Graph the individual functions
	+ The intersection points will stay the same.
	+ For the other points, find the y-values for the same x-value and add them. The sum will be the y-value of the combined function.
	+ Repeat this between each intersection at least twice to get an idea of the shape of the graph and connect the dots.
* Exercise 5.5
	+ Work on problems 25, 35, and 39 together as a class.
	+ Then have students go up to the whiteboards and work on the odd problems also have students use their electronic devices to graph some of the problems.
	+ Walk around, help students, and check on their answers.
 |
| Homework Assigned: | Assign homework at the end of class and give students a couple of minutes to get started on it.Day 1p. 88 Exercise 5.5 – by 3s (complete the divisible problem numbers by 3)Graph #3, 6, 15, and 18 by hand, and then use Desmos for #9, 12, 21, and 24. Do 2 graphs per screenshot.Day 2p. 89 Exercises 5.5 - by 3’s #27-33 complete by hand, #36-42 use Desmos and submit a screenshot (all 3 can be on 1 graph) |

 **5.6 – Graphs of Tangent, Cotangent, Cosecant, and Secant (2 Days)**

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| --- | --- |
| Start of Class: | Have students put solutions from yesterday’s assignment on the board. Students check the work on the board with their solutions. Ask for any specific problems they had while completing the homework that they need in more detail or show another solution. Have students turn in homework.Go over answers to Quiz 5.3-5.4 |
| Objectives: | * Know tangent is an odd function, and the reciprocal functions are the same even or odd function as the original.
* Graph the reciprocal trig functions
* Understand that asymptotes are lines that the graphs cannot touch or cross.
 |
| Lesson: | * Prove that tangent is an odd function
	+ Combining previous knowledge that
	+ Following this method, prove the reciprocal functions are even or odd.
* Graph
	+ Whenever , there is a problem. Cosine is in the denominator and the denominator can’t be 0. This creates an asymptote
	+ Asymptotes are invisible lines that we show as dotted lines. The graph will not touch or cross an asymptote.
* Work Examples 1 and 2
	+ Exercise 5.6
	+ Only have students work on odd problems that have tan or cot in them.
	+ Walk around, help students, and check on their answers.

Day 2* Graph cosecant and secant.
	+ Work off of sine and cosine graphs and the asymptotes.
	+ This covers Example 3
	+ Work Example 4
* Show students the [interactive unit circle](https://www.mathsisfun.com/algebra/trig-interactive-unit-circle.html)
	+ Now that students have seen all the graphs, this interactive unit circle shows how the sine, cosine, and tangent values translate to the graphed values.
	+ Once one revolution about the circle has occurred, the graphs repeat.
* Exercise 5.6
	+ Have students work on odd problems with sec or csc in them.
	+ Walk around, help students, and check on their answers.
 |
| Homework Assigned: | Assign homework at the end of class and give students a couple of minutes to get started on it.Day 1Give students Quiz 5.5 as a take-home quiz, they may use their notes and book. If completing the graph on Desmos, students need to copy the graph by hand onto their quiz paper.Day 2pp. 94-95 Exercises 5.6 - by 4’s #4, 8, and 12 - complete by hand, #16 and 20 use Desmos and submit a screenshot (both can be on 1 graph) |

 **5.7 – Periodic Motion – Application of Sine and Cosine (2 Days)**

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| --- | --- |
| Start of Class: | Have students put solutions from yesterday’s assignment on the board. Students check the work on the board with their solutions. Ask for any specific problems they had while completing the homework that they need in more detail or show another solution. Have students turn in homework.Go over answers to Quiz 5.5 |
| Objectives: | * Periodic functions are examples of simple harmonic motion
* Define frequency and know the unit is Hertz (Hz)
* Can calculate the frequency from the period and vice versa
* Can extract information from a word problem scenario to write a function and graph it.
 |
| Lesson: | * Show YouTube videos of pendulums and springs to show simple [harmonic motion](https://www.youtube.com/playlist?list=PLLmvDJR_FuZKd_2nwR8YY_mvpBIwww5sB)
* Read through scenarios given in the text about the rotating disk
	+ Relate the scenario to students riding on a merry-go-round and the speed they feel while sitting in the middle, halfway to the edge, and the outer rim.
	+ Graph Example 1 on Desmos and apply the slider feature for students to see how the graph changes when the distance from the center increases or decreases.
* Read through the spring scenario.
	+ Work on Example 3
	+ Show on Desmos with the slider feature to show how period and frequency are inversely related.

Day 2* Work on Exercise 5.7
	+ Do problems 1 and 7 together as a class.
	+ Have students pair up and complete the other problems on the board.
	+ Walk around, help students, and check on their answers.
 |
| Homework Assigned: | No homework these two days. |

**Review Days (1 or 2 days)**

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| --- | --- |
| Start of Class: | Hand out Review worksheets #1 and 2 for students to practice. |
| Lesson: | * Have students work on the board in pairs to complete the worksheets.
	+ Halfway through the class, students can switch to Desmos to check their graphs.
	+ Walk around, help students, and check on their answers.

Day 2 (if needed)* Give Quiz 5.6-5.7 – this quiz can be skipped if students are doing well.
* Work on Review Worksheet #3
	+ Have students pair up and complete the other problems on the board.
	+ Halfway through class, students can check their answers on Desmos.
	+ Walk around, help students, and check on their answers.
 |
| Homework Assigned: | Study for the Test |

**Test (1 Day)**

|  |  |
| --- | --- |
| Start of Class: | * Write the following 2 equations on the board. Students are to complete these 2 graphs in Desmos before they can have their test. (5 minutes)
* Hand out tests and go over directions
 |
| Lesson: | * Walk around the room to make sure that students are focused and keeping their eyes on their own paper.
* Students may work on other classwork after the test is handed in.
 |
| Homework Assigned: | No homework tonight |

**Supplementary Materials:**

* All paper copies of worksheets, quizzes, and tests can be found in my Google folder, [Technology Integration Project](https://drive.google.com/drive/folders/19otLP6k2JsxcuuJexzD_Cv8CvwaMID-A?usp=sharing).
* [Desmos](https://www.desmos.com/calculator) will be used for all electronic graphs for practice, homework, quizzes, and tests.
* YouTube videos will be used for two lessons
	+ 5.4 – [Amplitude and Frequency examples and sound](https://www.youtube.com/watch?v=L-q-KFwjwDs)
	+ 5.7 – [Simple Harmonic Motion Experiments](https://www.youtube.com/playlist?list=PLLmvDJR_FuZKd_2nwR8YY_mvpBIwww5sB)
* 5.6 - [An interactive unit circle](https://www.mathsisfun.com/algebra/trig-interactive-unit-circle.html)
* Quiz 5.1-5.3 will be completed online with [FlexiQuiz](https://www.flexiquiz.com/SC/N/8d0c520b-4f91-4646-a367-b597bc470a76)

**Evaluation of Learners:**

Students will complete 3-4 quizzes during the chapter and one test at the end. The quizzes will give preliminary results for the teacher and students to see if the students are learning and able to complete the graphs early or if more time needs to be devoted to a particular section before the test. One quiz will be completed online, and the rest will be completed by hand. Desmos will be used on the test.

 A test will be completed at the end of the chapter to test student learning and ensure that any areas of struggle are mastered before moving on to the next chapter.

**Evaluation of the Instruction:**

 If half of the class or more scores below 70 percent, then we will go back and work on the weak areas of the chapter. The instructor will also conduct a verbal survey when reviewing the tests to get feedback from students on what worked well with the technology and what could use some tweaks for next time.

**Reflection**

 I was trained to use the Abeka curriculum in college. Abeka provides the content, but teachers must supplement any technology into the curriculum. This project forced me out of my comfort zone to find outside technology to make this chapter more applicable to my students.

 YouTube is a good resource for demonstrations and labs since our small school does not have extra money for lab equipment. The frequency video will allow students to hear what happens when the period and frequency change (Murray, 2024, p. 2). And while I may not demonstrate real-life examples in class, YouTube creates a more authentic way for my students to engage and get involved with the class (Herrington, et al, 2014, p. 405). Desmos should be a great addition to the curriculum since most work is done on computers, and it will help the students learn how to input equations into a computer.

 I plan to use test and quiz scores to analyze the students’ learning and if the technology was beneficial for this chapter. Even though the grades will give me an overview of the students’ learning, I want include Oblinger’s advice to have a conversation with the students to get personal feedback on what went well, and what I may need to adjust in planning for next year’s class to keep current with the students’ needs (Oblinger, 2013, p. 49).

**References:**

Herrington, J., Reeves, T. C., & Oliver, R. (2014). Authentic Learning Environments. In *Handbook of Research on Educational Communications and Technology* (pp. 401–412). Springer New York. https://doi.org/10.1007/978-1-4614-3185-5\_32

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Oblinger, D. G. (2013). “Analytics: Changing the conversation”. *EDUCAUSE Review*, *48*(1), 48-49.